University of Zurich[UZH]

Agent-based Financial Economics
Lesson 5: World3 Model

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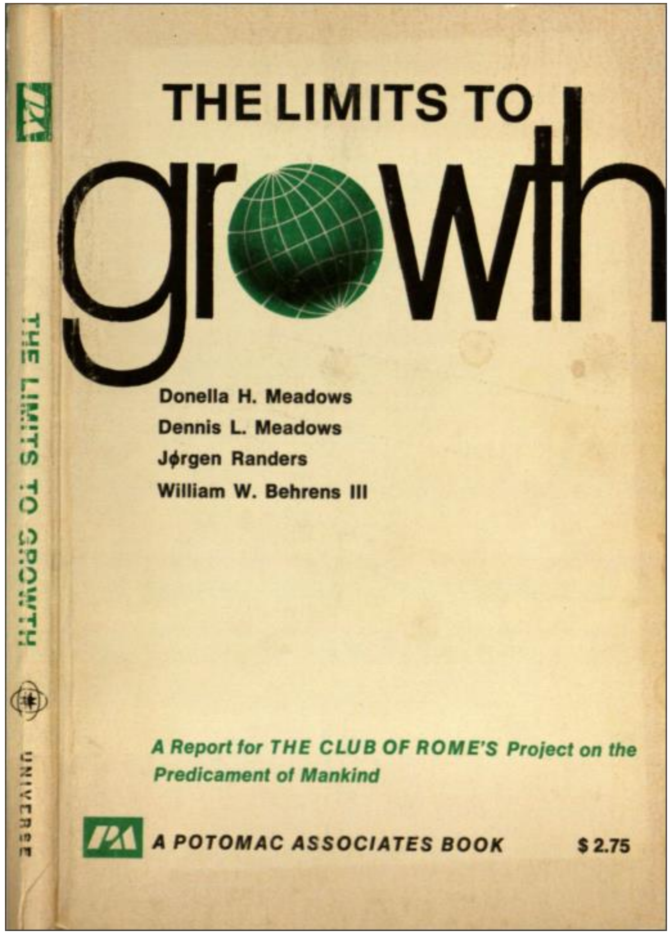
"What I cannot create, I do not understand."

- Richard Feynman

Today

- Discussion of exercise 3
- Club of Rome Model
- Exercise 4: demographics

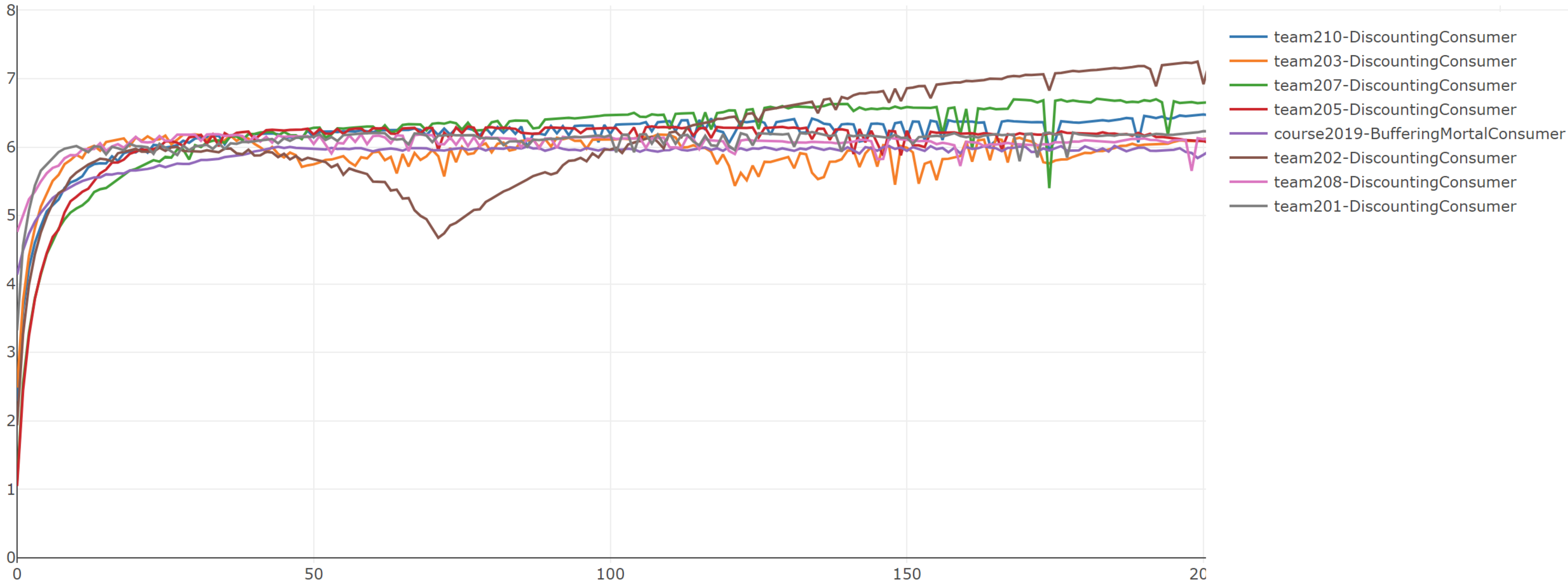


Exercise 3: Discussion

Ranking

1	team205-DiscountingConsumer	6.056087487	START = 0.9, STEP = 0.0002
2	team202-DiscountingConsumer	6.037691144	START = 0.9, STEP = 0.001, many variants tried
3	team201-DiscountingConsumer	5.944594389	START = 0.82, Step proportional to difference, Println!
4	team208-DiscountingConsumer	5.939114961	START = 0.92, STEP = 0.001, DISCOUNT = 2.7%, ,
5	team210-DiscountingConsumer	5.88886249	START = 0.92, STEP depending on difference and age
6	team203-DiscountingConsumer	5.812440183	START = 0.9, STEP depending on spendings and others
7	team207-DiscountingConsumer	5.757272077	START = 0.95, adjustable speed
8	course2019-BufferingMortalConsumer	5.743647431	START = 0.9, no adjustment

Exercise 3: Discussion



Exercise 3: Optimal Savings

Some teams have asked where the buffer size heuristic comes from.

First: note that log utility implies to distribute spending according to the utility weights.

$$U = \sum_{i=1}^{n} \alpha_i \log x_i \quad \text{s.t.} \quad \sum x_i p_{x,i} = W$$

$$\Rightarrow \quad w_k = \frac{\alpha_k}{\sum_{i=1}^{n} \alpha_i} W \quad \Rightarrow \quad x_k = \frac{w_k}{p_{x,k}}$$

Exercise 3: Optimal Savings

Start with discounted utility maximization and assume interest r on what is saved for the future:

$$\max U = \sum_{t=0}^{\infty} \beta^t u_t = \sum_{t=0}^{\infty} \beta^t \ln (1+r)^t x_t = \sum_{t=0}^{\infty} \beta^t \ln x_t + \underbrace{\sum_{t=0}^{\infty} \beta^t \ln (1+r)^t}_{\text{constant}}$$

$$\text{subject to: } \sum_{t=0}^{\infty} x_t = W = \sum_{t=0}^{\infty} \frac{h \cdot p_h}{(1+r)^t} = \frac{h \cdot p_h}{r}$$

Example: $r = 1\%$, $h = 24$, $p_h = 1$ $\Rightarrow$ $W = 2400$

$\beta = 0.99 \Rightarrow x_0 = \frac{W}{100} = 24 \Rightarrow$ no savings when $\beta = 1 - r$

(Made use of geometric sum equation for some transformations.)

(Sloppy assumption: $\frac{1}{1+r} = 1 - r$, holds approximately for small r)

Exercise 3: Optimal Savings

Leads to a simple heuristic for how much to save:

Generally: $x_0 = (1-\beta)\,\omega = (1-\beta)\frac{1}{r} h \cdot p_h = \frac{\delta}{r} h p_h$ (with $\delta = 1-\beta$)

Example: When $\delta = 1\%$ and $r = 3\%$, $\frac{1}{3} \cdot 24 = 8$ hours can be consumed, everything else must be saved, whereas "saving 16 hours" means working that much and put the money aside.

Underlying assumption: wages p_h are constant.
Potatoe prices can fluctuate.

Exercise 3: Optimal Savings

But wait, on day 1, we might have some savings from day 0!

$$x_t = (1-\beta)W_t = \underbrace{\delta}_{\text{e.g. } 1\%}\Big(\underbrace{\tfrac{1}{r} hp_h}_{\text{net present value of all man-hours}} + \underbrace{(1+r) s_{t-1}}_{\text{savings}}\Big)$$

⇒ In equilibrium, spend 1% of savings and all 24 hours

⇒ 99% buffer rule

$$s_t = \underbrace{\beta(1+r) s_{t-1}}_{\text{what was kept from the previous savings}} + \underbrace{\left(hp_h - \tfrac{\delta}{r} hp_h\right)(1+r)}_{\text{what was saved from the wage (or dissaved)}}$$

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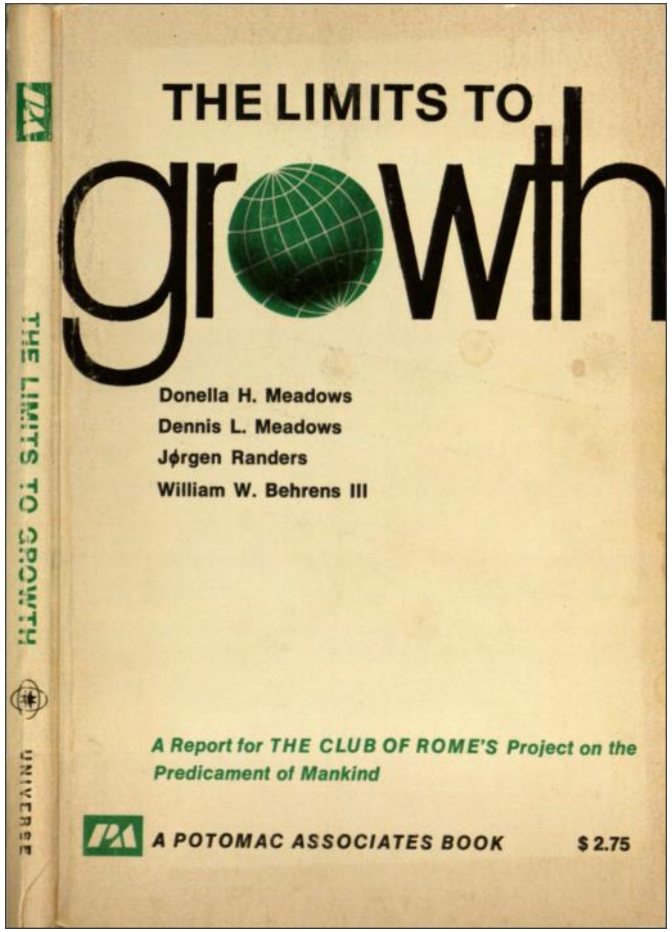
→ It becomes apparent that the 99% is a rule that leads to a stable steady state. Once the steady state is reached, the rule is optimal, but it does not provide the optimal path to get there.

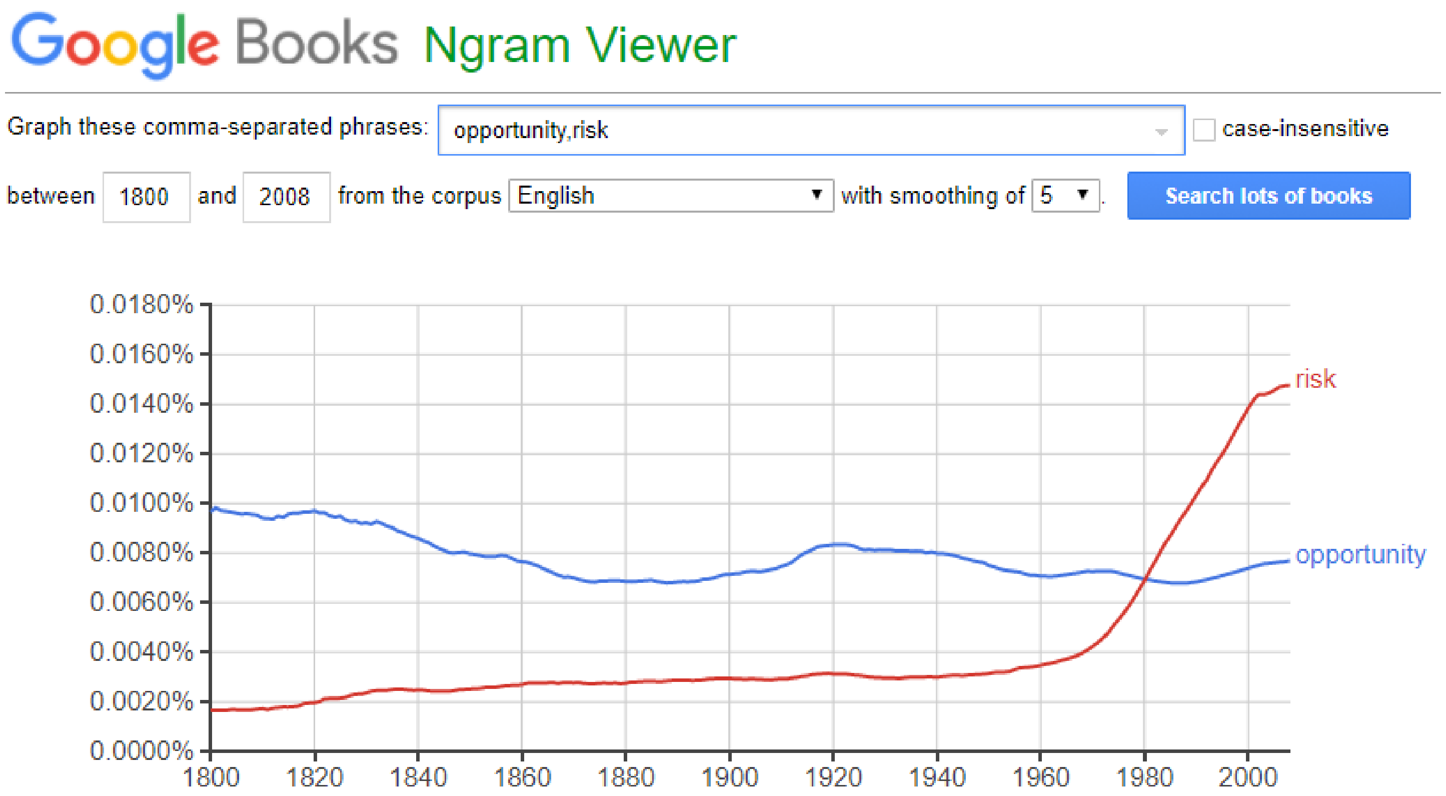
→ Initially, the agent should put aside less then 99%, depending on the interest rate.

Club of Rome: Limits to Growth



- Hugely influential book from 1972
- Based on System Dynamics (not agent-based, but also exhibits non-linear endogenous dynamics)
- Start of the green movement: recycling, outlawing DDT, etc.
- Pessimistic predictions
- PDF available from: www.clubofrome.org/report/the-limits-to-growth

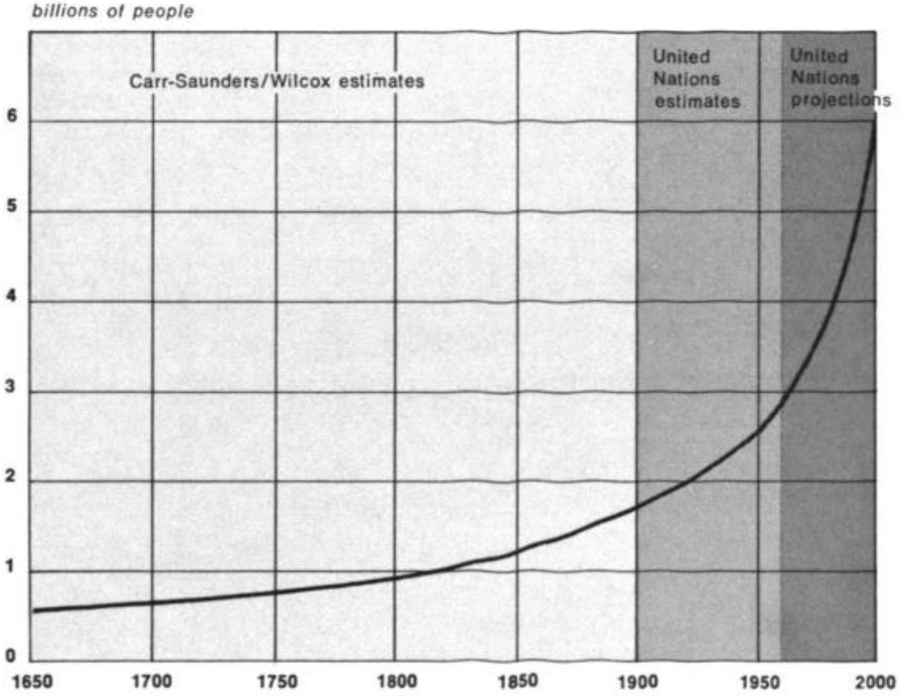
Club of Rome: Limits to Growth



Caused a paradigm shift: awareness that we can destroy the planet.

Club of Rome: Limits to Growth

Figure 5 WORLD POPULATION

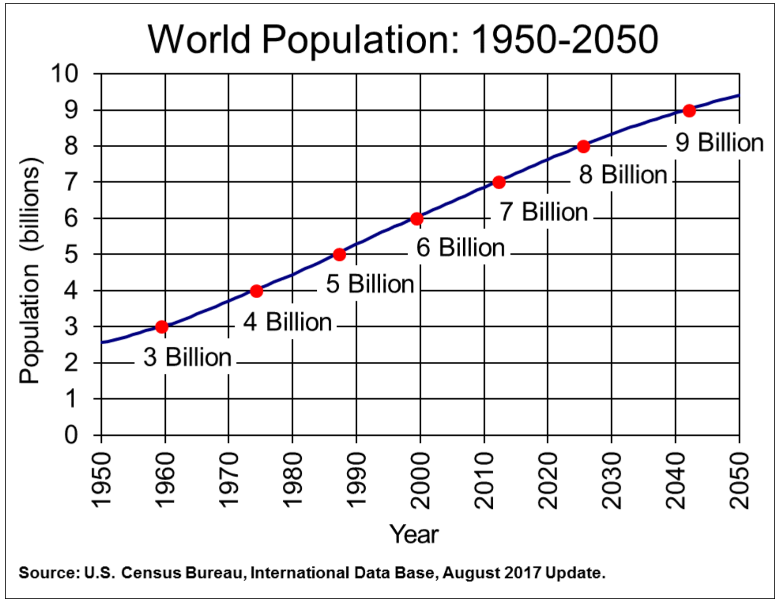


World population since 1650 has been growing exponentially at an increasing rate. Estimated population in 1970 is already slightly higher than the projection illustrated here (which was made in 1958). The present world population growth rate is about 2.1 percent per year, corresponding to a doubling time of 33 years.

SOURCE: Donald J. Bogue, *Principles of Demography* (New York: John Wiley and Sons, 1969).

Some estimates have been excellent.
Prediction for world population in the year 2000 has been spot on.

Current outlook



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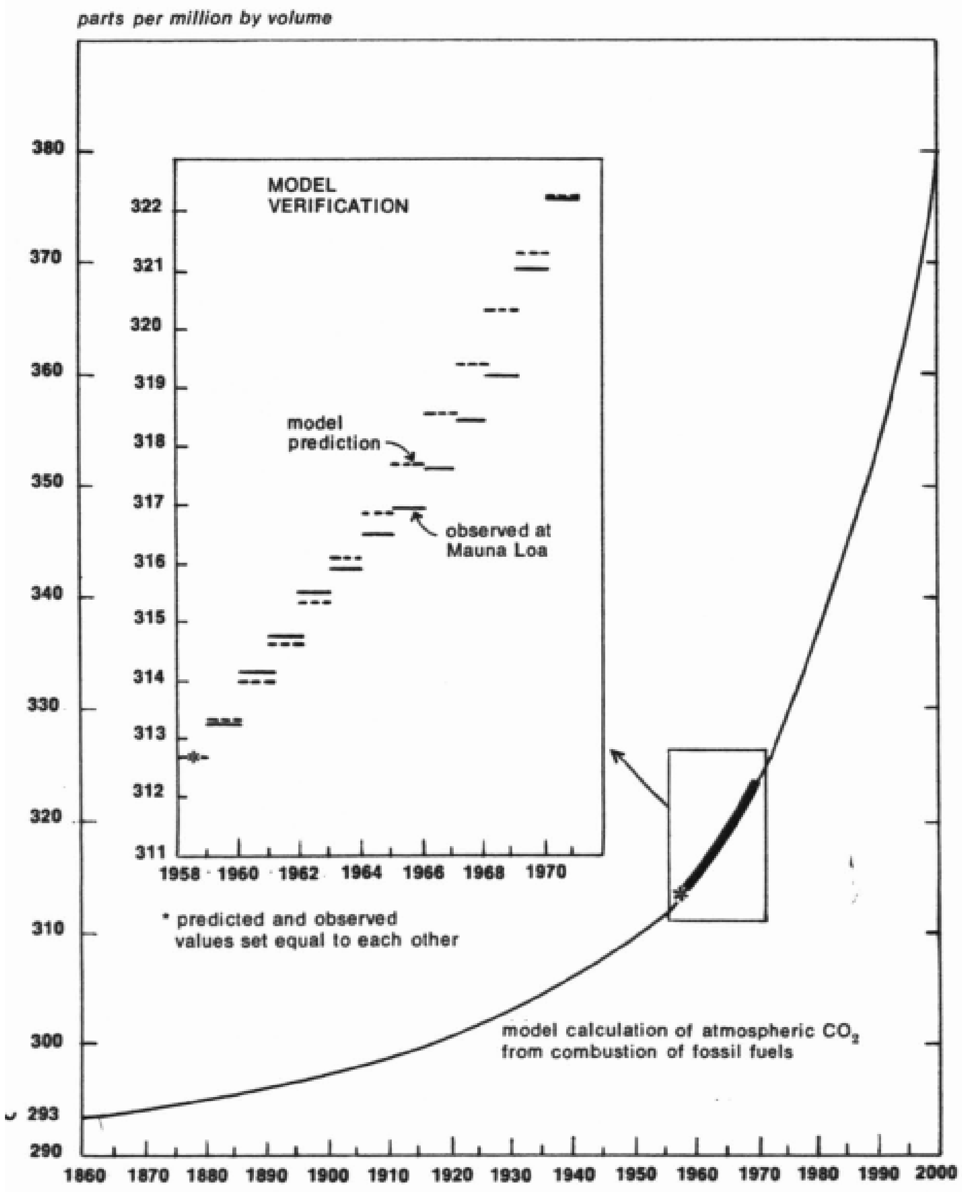
Also prediction for CO_2 concentration in atmosphere was excellent.

Current level: around 400 ppm

Figure 15 CARBON DIOXIDE CONCENTRATION IN THE ATMOSPHERE



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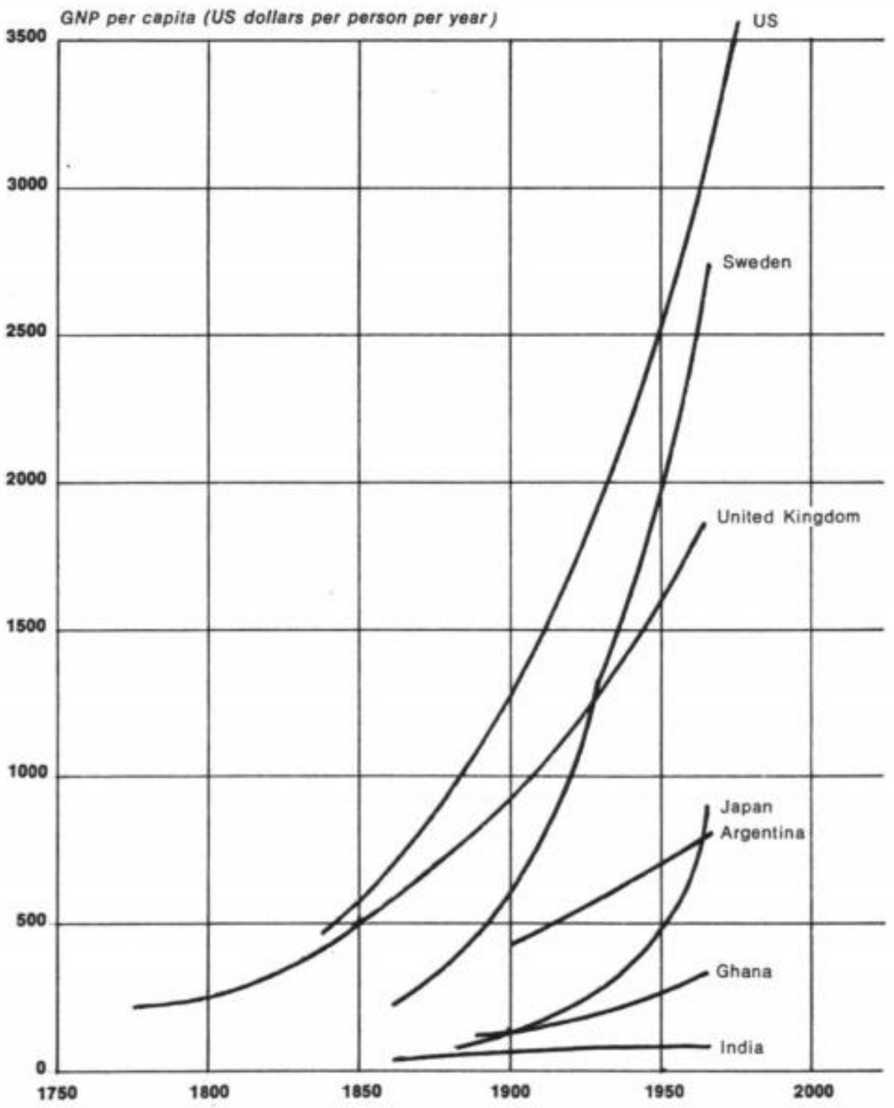
Basic observation: things are growing exponentially.

Table 2 ECONOMIC AND POPULATION GROWTH RATES

Country	*Population (1968) (million)*	*Average annual growth rate of population (1961–68) (% per year)*	*GNP per capita (1968) (US dollars)*	*Average annual growth rate of GNP per capita (1961–68) (% per year)*
People's Republic of China *	730	1.5	90	0.3
India	524	2.5	100	1.0
USSR *	238	1.3	1,100	5.8
United States	201	1.4	3,980	3.4
Pakistan	123	2.6	100	3.1
Indonesia	113	2.4	100	0.8
Japan	101	1.0	1,190	9.9
Brazil	88	3.0	250	1.6
Nigeria	63	2.4	70	—0.3
Federal Republic of Germany	60	1.0	1,970	3.4

Figure 7 ECONOMIC GROWTH RATES



The economic growth of individual nations indicates that differences in exponential growth rates are widening the economic gap between rich and poor countries.

SOURCE: Simon Kuznets, *Economic Growth of Nations* (Cambridge, Mass.: Harvard University Press, 1971).

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Basic observation: things are growing exponentially.

What if we extrapolate this?

Table 3 EXTRAPOLATED GNP FOR THE YEAR 2000

Country	*GNP per capita (in US dollars *)*
People's Republic of China	100
India	140
USSR	6,330
United States	11,000
Pakistan	250
Indonesia	130
Japan	23,200
Brazil	440
Nigeria	60
Federal Republic of Germany	5,850

* Based on the 1968 dollar with no allowance for inflation.

1 USD from 1968 corresponds to 7 USD from 2017.

US estimate is okayish (57k vs 77k). Others are way off.

Actual vs Club of Rome estimate:
China: 8k vs 0.7k → Underestimated China
Russia: 9k vs 42k → Overestimated Russia
Japan: 39k vs 160k
Nigeria: 2.2k vs 0.4k
Germany: 42k vs 42k
Brazil: 8.6k vs 3k
Indonesia (now 3.5k) overtook Pakistan (now 1.5k)

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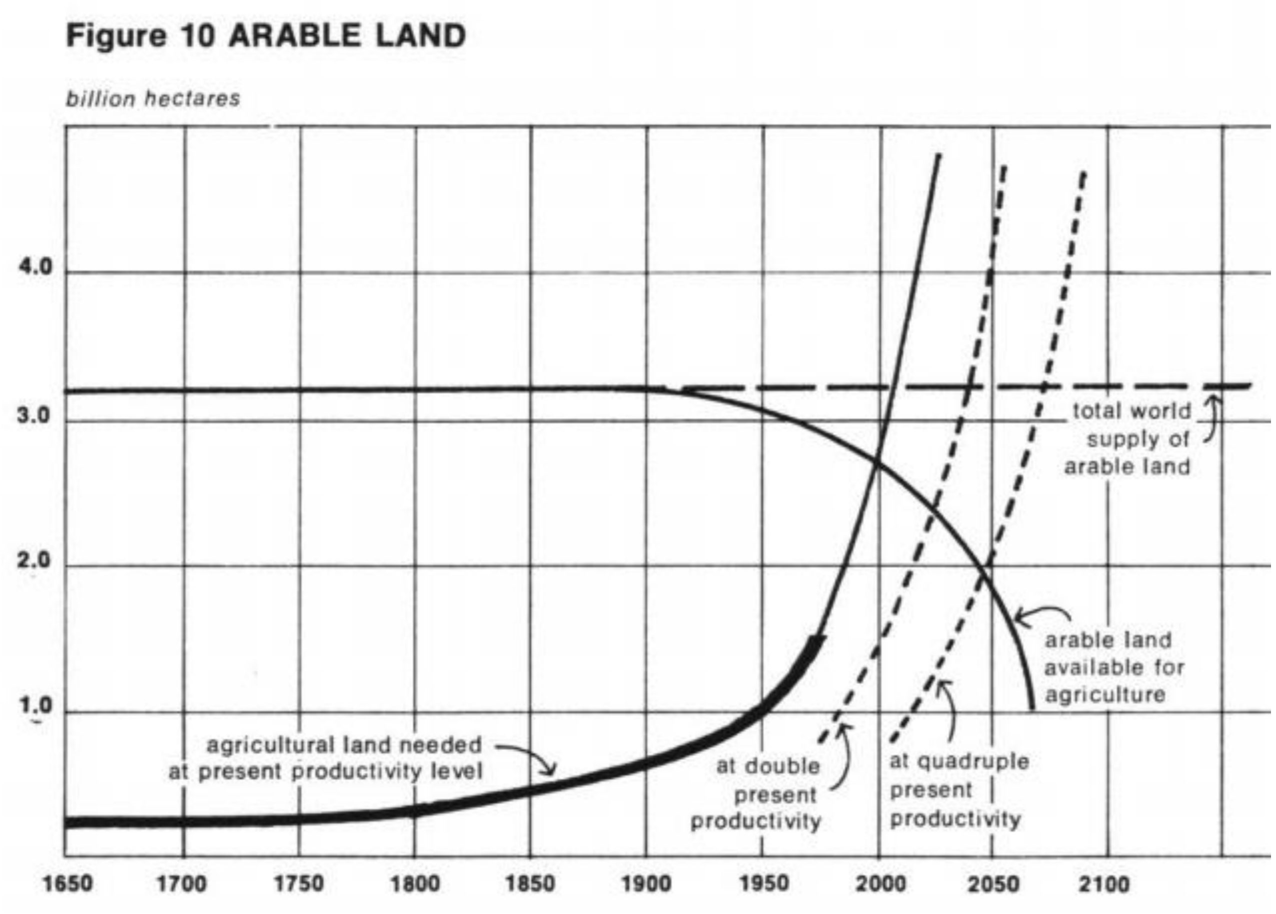
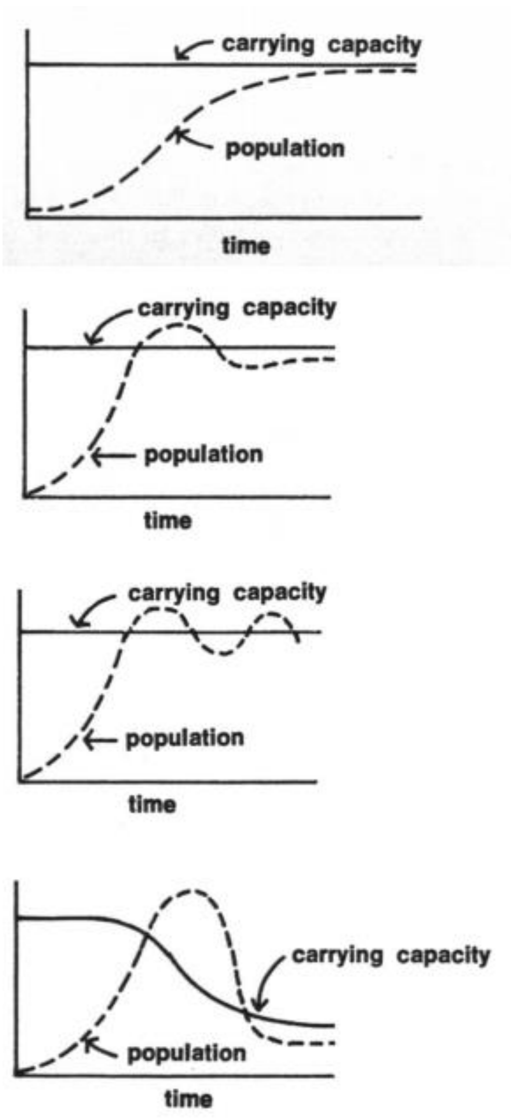
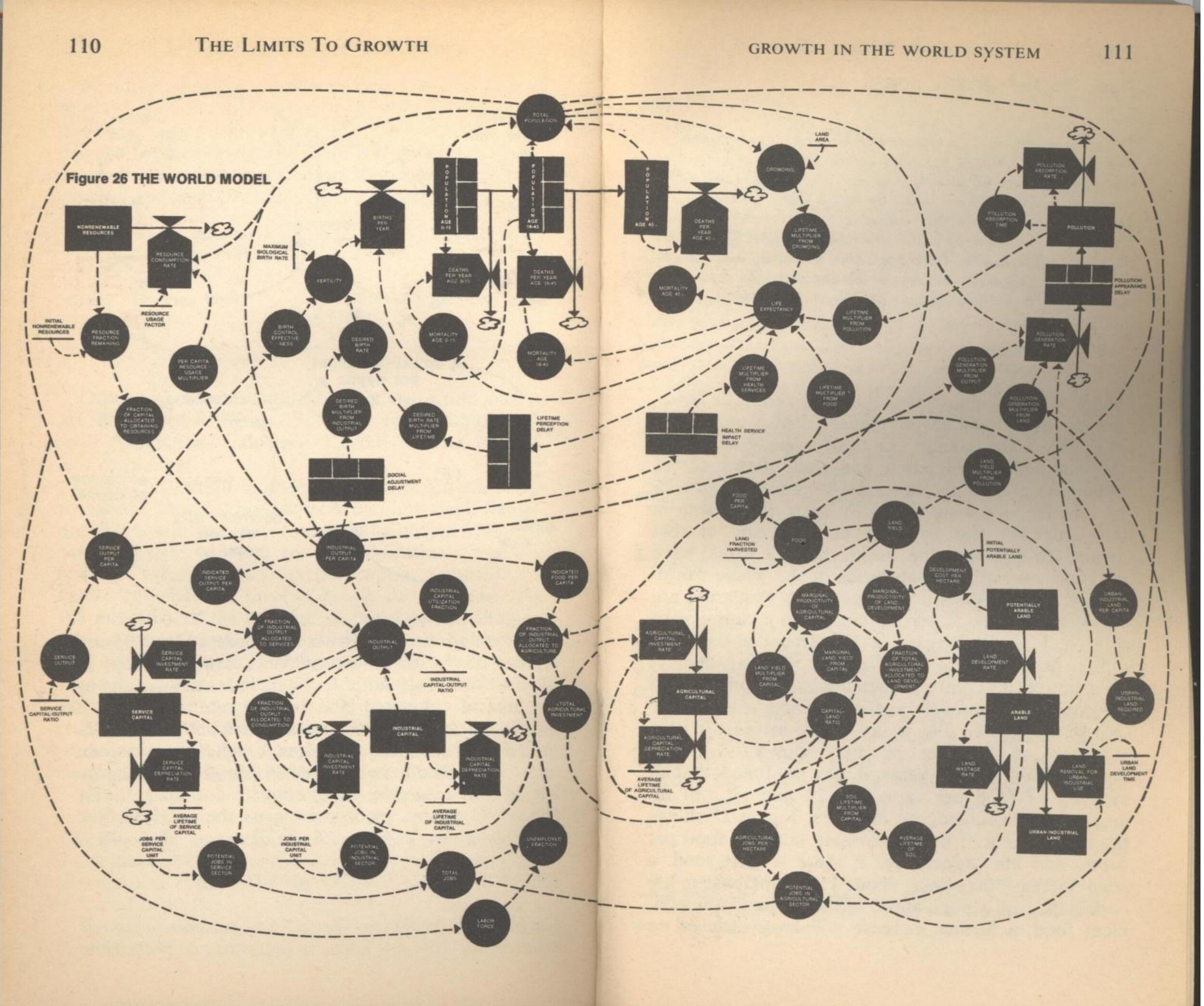


Club of Rome warning: Regardless of how accurate our predictions are, with exponential growth, we will hit some natural limits sooner or later! This cannot go on forever!



Types of dynamics.



The “Limits to Growth” world model.

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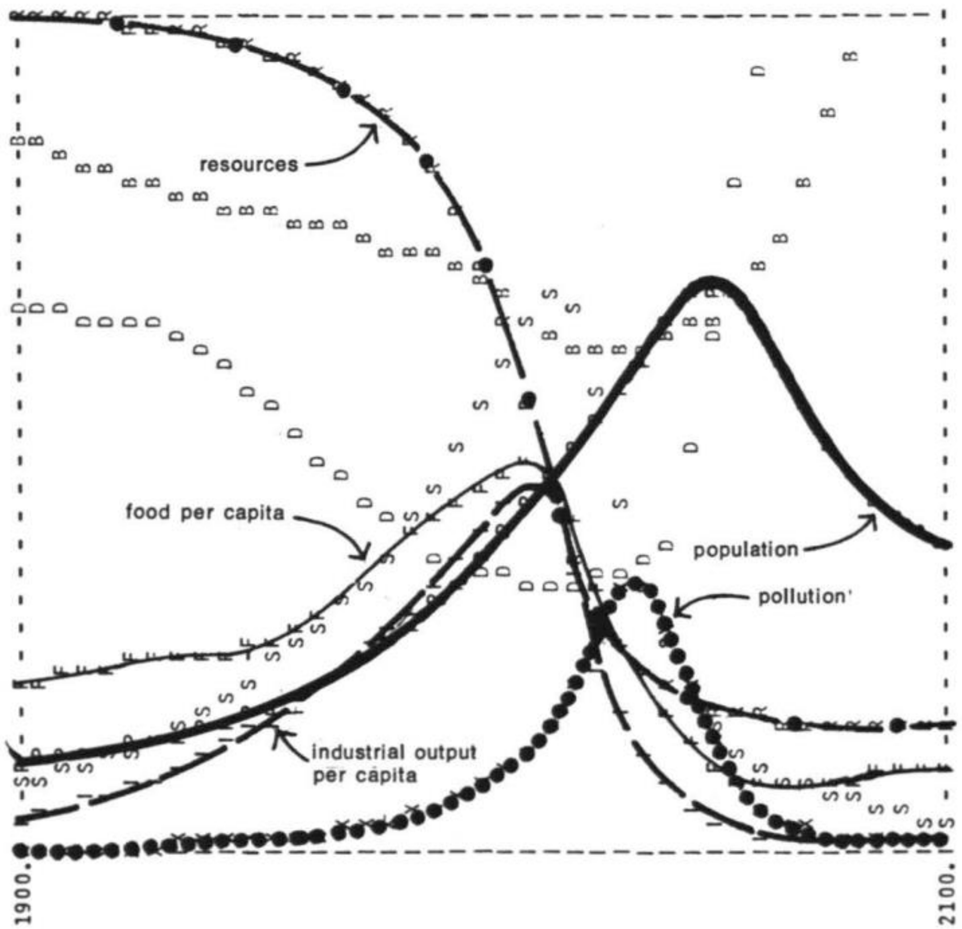
Figure 35 WORLD MODEL STANDARD RUN



The "standard" world model run assumes no major change in the physical, economic, or social relationships that have historically governed the development of the world system. All variables plotted here follow historical values from 1900 to 1970. Food, industrial output, and population grow exponentially until the rapidly diminishing resource base forces a slowdown in industrial growth. Because of natural delays in the system, both population and pollution continue to increase for some time after the peak of industrialization. Population growth is finally halted by a rise in the death rate due to decreased food and medical services.

→ Turned out to be overly pessimistic. Underestimated inventiveness of firms and free innovation, i.e. adjustment to less resource usage as they got more expensive. Did not foresee the "digital age". Instead, they called for the creation of "supranational institutions" to manage population and capital growth…

You can play with the model online on:
insightmaker.com/insight/1954/The-World3-Model-A-Detailed-World-Forecaster

Possible Seminar Work

Outlook:

- Manage and program an investment fund in our simulated world
- Analyze the World3 Model in more detail, try to update it and present the results.
- Do the same for another model of your choice.

→ Let me know in the next lecture what your preferred option is

Presentations

- 1.11., 8.11., 15.11., 22.11: normal lessons, refining our model, experimenting
- 6.12. Presentations of three or four teams (the topic teams)
- 13.12. Presentations of three or four teams (the simulation teams)
- 20.12. Special smart contracts lesson

Model Adjustments

Now:

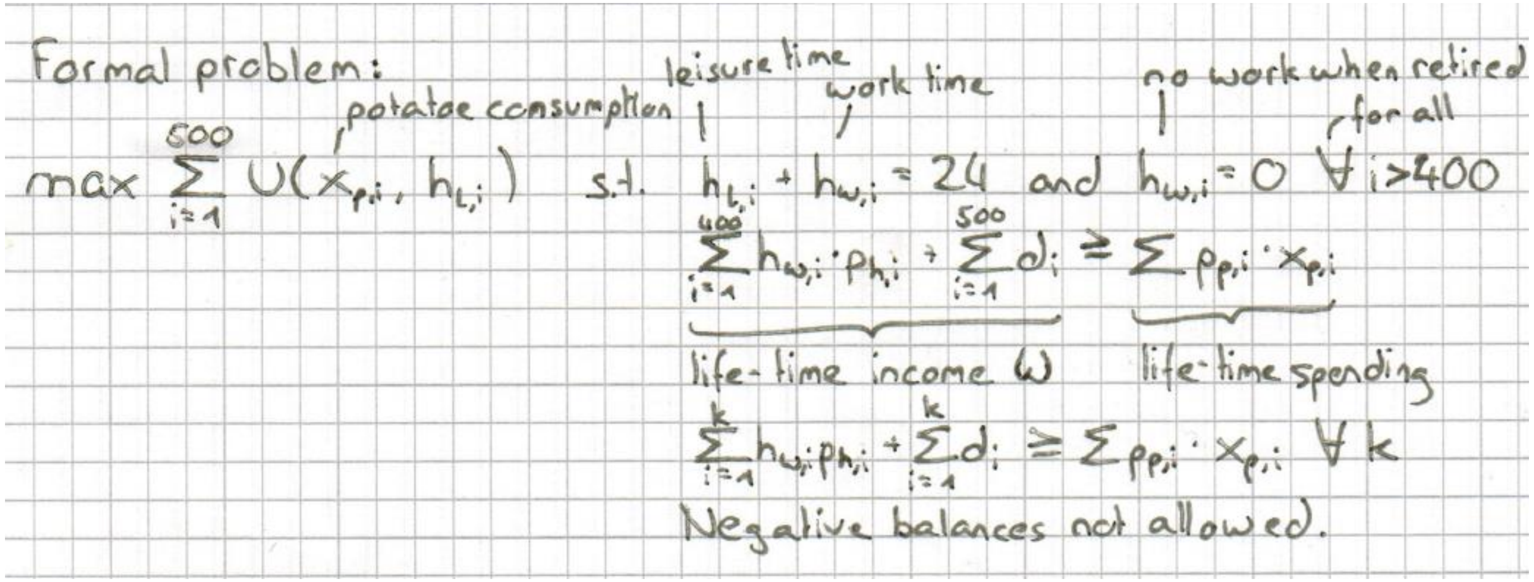
- Agents get fixed life-span of 250 days (→ no more discounting)
- Agents retire at age 200 and stop working
- We drop the fixed costs in the production function for better stability

Outlook:

- The stocks of all companies are freely tradable
- Agents invest in investment funds, no interest on money any more
- Investment funds are managed by you

Savings Heuristics

Formal problem looks complicated, with lots of variables and unknowns…



(Calculations are down with life expectancy of 500 and retirement at age 400 here.)

Savings Heuristics

If prices and wages are constant, the problem simplifies to:

$max \sum_{i=1}^{500} u(x_{p,i})$ subject to the budget constraint $\sum_{i=1}^{500} px_{p,i} = \sum_{i=1}^{400} w_i = 400w$ (without dividends for now)

It is optimal to smooth consumption, and to consume the same number of potatoes every day. But what if prices can change?

Change in potatoe price has no effect, as "consumption smoothing" with log utility is in fact "expenses smoothing", i.e. the same amount gets spent on consumption goods every day, regardless of their prices. However, varying wages make a difference as they change the net present value of our life-time income.

Savings Heuristics: But what about interest?

Adding interest rates does not change anything either.
The income effect tells me: “Save money today, so you can spend even more on potatoes tomorrow.”
The substitution effect tells me: “You can spend more today, thanks to interest your money will grow back.”
→ Both effects cancel out, and I still decide to spend the same amount today.
(More precisely, if I previously spent 100 on day one and 100 on day two, introducing an interest rate of 10% does not affect my spending on day one, but I will spend 110 on day two.)

What about interest?

Consider simple two-period model $U(x_1, x_2) = \log x_1 + \log \overbrace{(1+r)}^{\text{interest}} x_2$

s.t. $x_1 + x_2 = W$

$\Rightarrow U(x_1, x_2) = \log x_1 + \log x_2 + \underbrace{\log(1+r)}_{\text{constant!}}$

$\Rightarrow$ still spend the same amount on day 1.

Savings Heuristic for Retirees

These considerations lead us to a very simple, but also very effective decision heuristic for retirees:

Simply spend 1/d of your wealth today if you have d days left to live.

This heuristic is robust against:

- Nominal and real price changes
- Inflation / deflation
- Changes in nominal and real interest rate
- Dividends (work like interests), when stocks can be sold
- Mispricing of stocks

Caveat:

- It only works so nicely thanks to assuming log-utility.

Savings Heuristic for Retirees

Thus, the implementation for the retiree could look as follows:

```
public void managePortfolio(IStockMarket stocks) {
    boolean retired = isRetired();
    if (retired) {
        int daysLeft = getMaxAge() - getAge() + 1;
        double consumptionToday = this.savings / daysLeft;
        this.savings -= consumptionToday;
    } else {
```

Savings Heuristic for Workers

(Still disregarding interest and dividends in the optimization.)

In order to spend the same amount every day, about 1/5 of the daily work income needs to be saved, and 4/5 can be spent on potatoes. In other words: if daily spendings are 100, an amount of 25 should go into savings.

```
public void managePortfolio(IStockMarket stocks) {
    boolean retired = isRetired();
    if (retired) {
        int daysLeft = getMaxAge() - getAge() + 1;
        double consumptionToday = this.savings / daysLeft;
        this.savings -= consumptionToday;
    } else {
        double dividends = getPortfolio().getLatestDividendIncome(); // how much dividends did we get today?
        double workFraction = 1.0d / getMaxAge() * getRetirementAge(); // 80%
        double retirementFraction = 1 - workFraction; // 20%
        this.savings += (getDailySpendings() - dividends) / workFraction * retirementFraction;
    }
}
```

→ E.g. equation tells consumer to put aside 20$ per day. If the consumer worked 10 hours before earning 100$, he will now work e.g. 11 hours earning 110$, put 20$ aside and buy potatoes worth 90$.

Savings Heuristic for Workers with Interest

To behave optimally, the agent should spend an equal share of his life-time wealth W_tot every day.

Now, the interest (or dividends) make a difference! They define how much future work income should be discounted and thus also what our net present wealth is. The fact that the agent does not discount the future utility any more does not matter here.

→ save more when interest is high, even with log utility and no discounting
(Since value of W depends on interest)

Savings Heuristic for Workers with Interest

Last years heuristic assumed constant wage income, which is too simplistic.
→ Still using it for now, but plan to refine it and discuss it then.

Using a simple two-period example with variable leisure time to show that agents actually should work harder in the first period.

$$\max_{x_0, x_1, h_0, h_1} \log(x_0) + \log(24-h_0) + \log(x_1) + \log(24-h_1) \quad \text{s.t.} \quad wh_0 + \frac{wh_1}{(1+r)} = x_0 + \frac{x_1}{(1+r)}$$

$$L: \log(x_0) + \log(24-h_0) + \log(x_1) + \log(24-h_1) + \lambda\left(wh_0 + \frac{wh_1}{(1+r)} - x_0 - \frac{x_1}{(1+r)}\right)$$

$$\frac{\partial L}{\partial x_0}: \frac{1}{x_0} + \lambda(-1) = 0 \Rightarrow \lambda = \frac{1}{x_0}$$

$$\frac{\partial L}{\partial h_0}: \frac{-1}{24-h_0} + \lambda w = 0 \Rightarrow \lambda = \frac{1}{(24-h_0)w}$$

$$\frac{\partial L}{\partial x_1}: \frac{1}{x_1} + \lambda\frac{-1}{1+r} = 0 \Rightarrow x_0(1+r) = x_1$$

⇒ eats less today than tomorrow

$$\frac{\partial L}{\partial h_1}: \frac{-1}{24-h_1} + \lambda\frac{w}{1+r} = 0 \Rightarrow (24-h_0)(1+r) = (24-h_1) \Rightarrow h_1 = 24-(24-h_0)(1+r)$$

⇒ works more today than tomorrow

Exercise 4

See online.